4.4 Sine and Cosine: with any angle.

Trigonometry

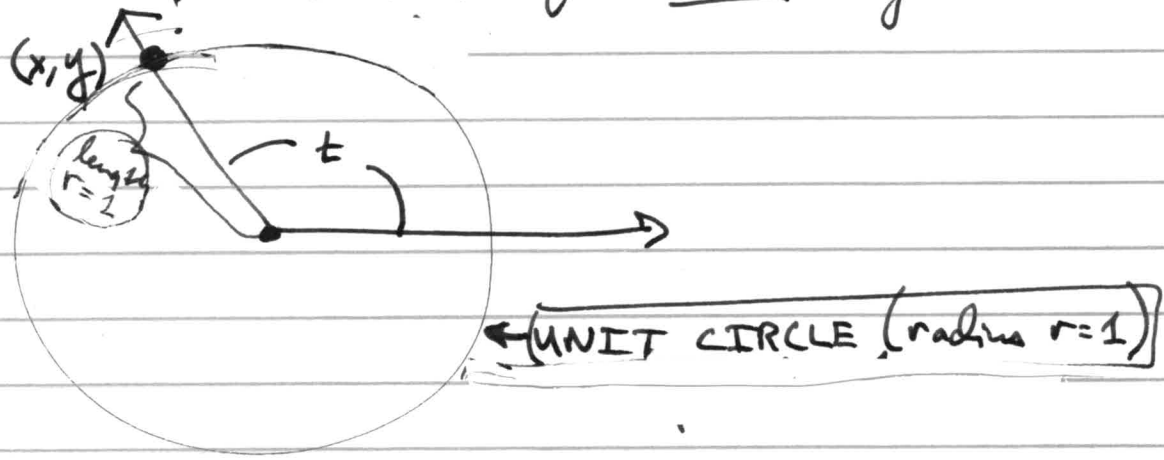
1060 - Day 15

Fall 2013

Last time, we used right triangles to compute things like $\sin\left(\frac{\pi}{4}\right)$.

But this definition only worked for $0 < \theta < \frac{\pi}{2}$

We now define $\sin(t)$ for ANY angle t



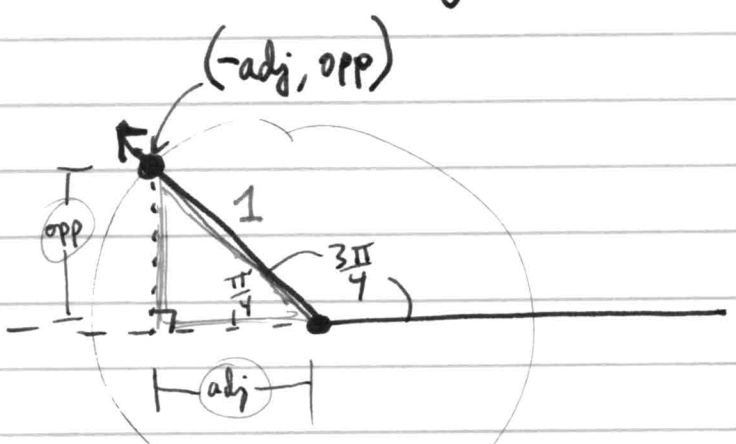
Define:

$$\sin(t) = \frac{y}{r} = y$$

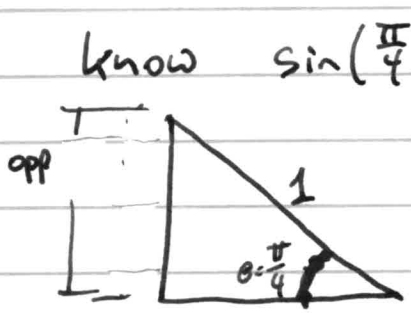
because $r=1$ on the unit circle

$$\cos(t) = \frac{x}{r} = x$$

[redacted]. find $\sin\left(\frac{3\pi}{4}\right)$ and $\cos\left(\frac{3\pi}{4}\right)$



know $\sin\left(\frac{\pi}{4}\right)$ and $\cos\left(\frac{\pi}{4}\right)$



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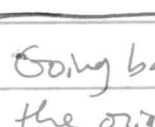
$$\sin\left(\frac{\pi}{4}\right) = \frac{opp}{hyp} : \quad = \frac{opp}{1} = opp$$

and

$$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

so: $opp = \frac{\sqrt{2}}{2}$

Going back to the original pic



Notice: the y-value is positive

so $\sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$

& $\cos\left(\frac{\pi}{4}\right) = \frac{adj}{hyp} = adj = \frac{\sqrt{2}}{2}$

so $adj = \frac{\sqrt{2}}{2}$

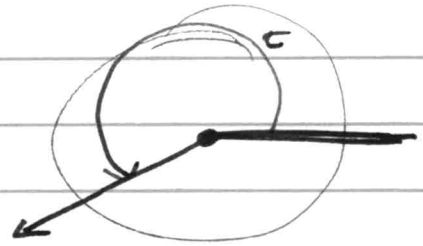
Going back to orig pic

Notice: the x-value is negative

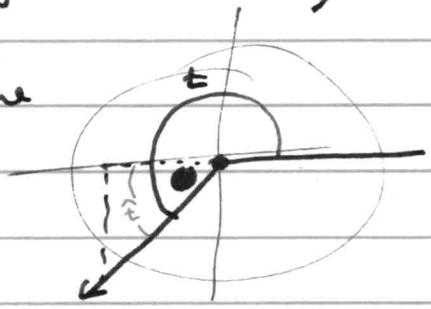
so $\cos\left(\frac{3\pi}{4}\right) = \frac{-\sqrt{2}}{2}$

In general, to find sine & cosine of a big angle t

① Sketch the angle and find the Δ it makes (by adding the missing vertical side)



② find the lengths of the horiz & vertical sides using Δ, tri's



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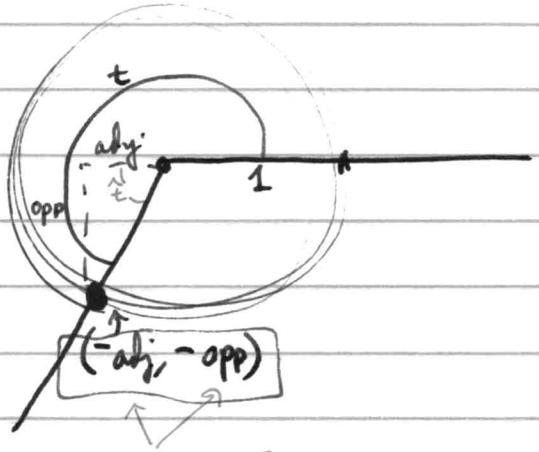
Define: the inside $\measuredangle t$ is called the **reference angle**

③ look back at the original pic, & decide if sine and/or cosine of t should be negative.



Both sine & cosine are negative

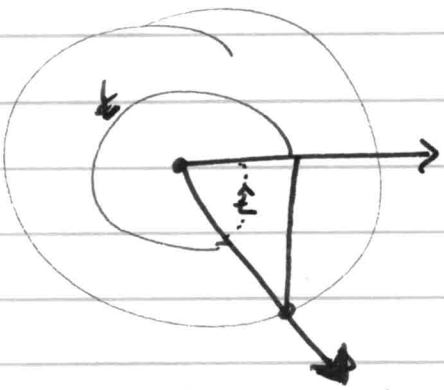
textbook calls this r

I prefer $\hat{t}$ = (t wearing a fancy hat)

The Reference Number (helper angle)

is

Picture



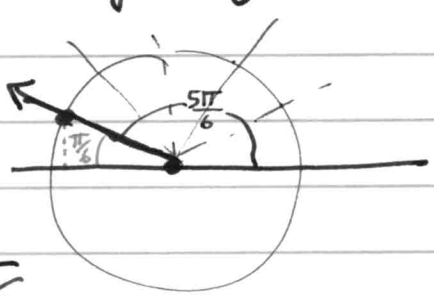
words

the smallest $\measuredangle$ that gets you back to the x-axis

the angle in the △ you get by filling in the missing side

Eg: find the reference Angle for

$t = \frac{5\pi}{6}$



Notice

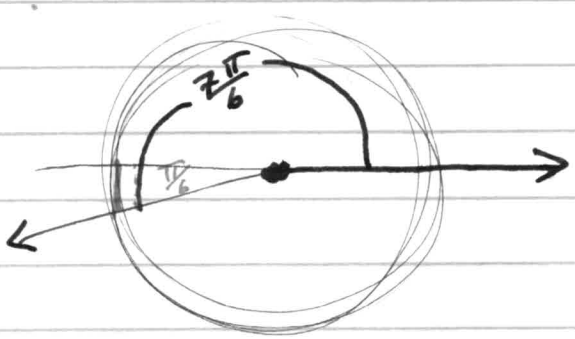
$\frac{5\pi}{6}$ + helper angle = π

so

helper angle = $\frac{\pi}{6}$

helper angle $\hat{t} = \frac{\pi}{6}$

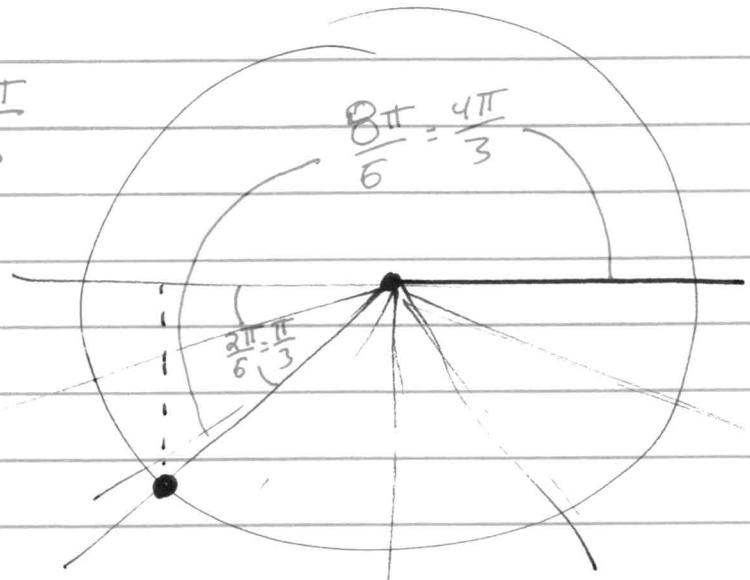
$t = \frac{7\pi}{6}$



helper angle $\hat{t} = \frac{\pi}{6}$

$t = \frac{8\pi}{6} = \frac{4\pi}{3}$

helper $\measuredangle \hat{t} = \frac{\pi}{3}$



Notice: $\left(\frac{4\pi}{3} = \pi + \text{helper angle}\right) \Rightarrow \left(\text{helper angle} = \frac{\pi}{3}\right)$

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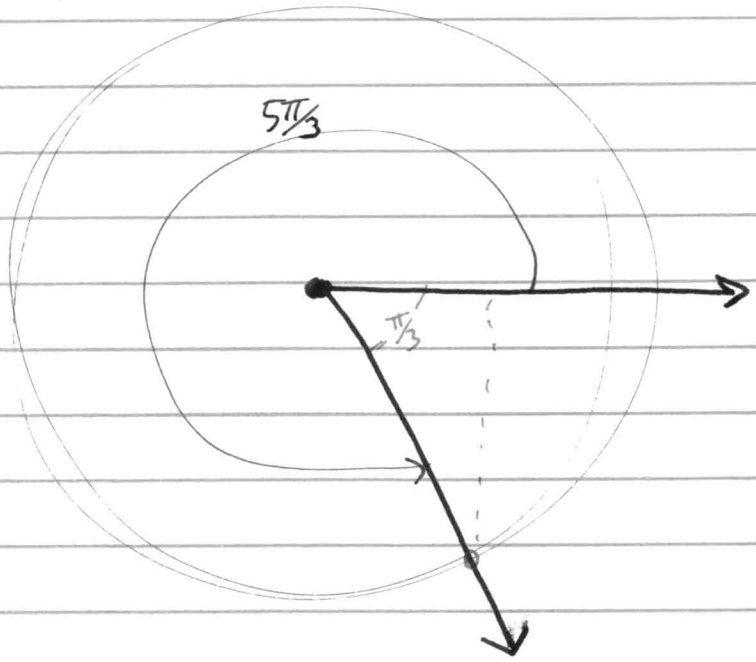
$t = \frac{5\pi}{3}$

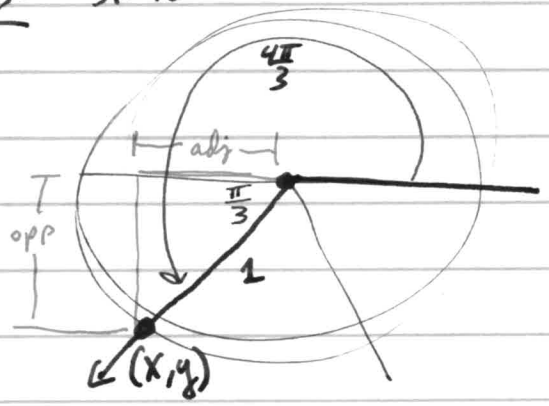


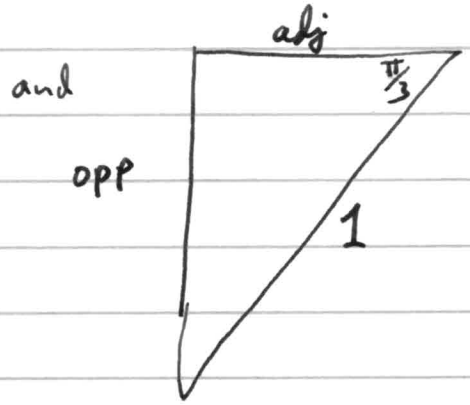
Notice $\frac{5\pi}{3} + \text{helper angle} = 2\pi$

so $\boxed{\text{helper angle} = \frac{\pi}{3}}$

Eg: Compute $\sin\left(\frac{8\pi}{6}\right) = \sin\left(\frac{4\pi}{3}\right)$

first sketch





$\sin\left(\frac{\pi}{3}\right) = \frac{opp}{1} = opp$

&

$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

so $opp = \frac{\sqrt{3}}{2}$

the y-coordinate is negative

$\Rightarrow \sin\left(\frac{4\pi}{3}\right) = \frac{-\sqrt{3}}{2}$

$\cos\left(\frac{\pi}{3}\right) = \frac{adj}{1} = adj$

&

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

(so)

$adj = \frac{1}{2}$

the x-coordinate is negative

$\Rightarrow \cos\left(\frac{4\pi}{3}\right) = \frac{-1}{2}$